

Composition and Zero-Shot Transfer with Lattice Structures in Reinforcement Learning

Geraud Nangue Tasse*, Steven James, Benjamin Rosman

University of the Witwatersrand, Johannesburg, South Africa



RLC 2025

provably!!!

UNIVERSITY OF THE
WITWATERSRAND
JOHANNESBURG



MIND
Machine Intelligence and
Neural Discovery Institute



JAIR

*Journal of
Artificial Intelligence
Research*

Motivation

We want **instructable agents** (e.g. via language) that can **solve tasks beyond Boolean rewards** [1,2], and **generalise compositionally** to new tasks.



- “**Serve breakfast** with **plain toast** *and* **ketchup...**”
- Neural networks **struggle to generalize compositionally**³

[1] G. Nangue Tasse et al., “A Boolean task algebra for reinforcement learning” NeurIPS, 2020.

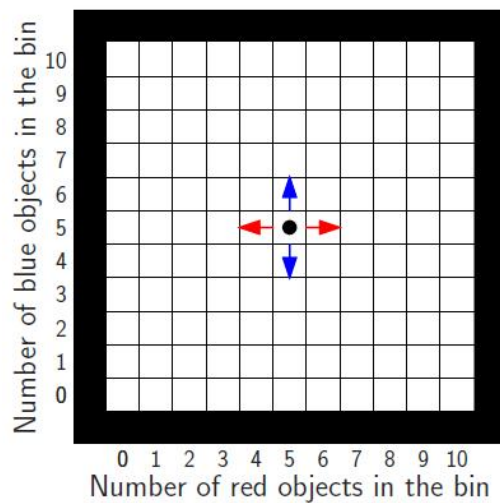
[2] R. T. Icarte et al., “Using reward machines for high-level task specification and decomposition in reinforcement learning,” ICML, 2018

[3] B. M. Lake et al., “Generalization without systematicity: On the compositional skills of sequence-to-sequence recurrent networks. PMLR 2018.

Motivation



Bin packing domain

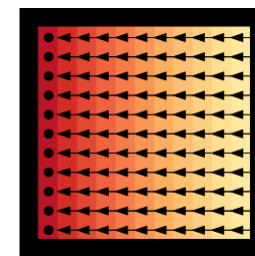
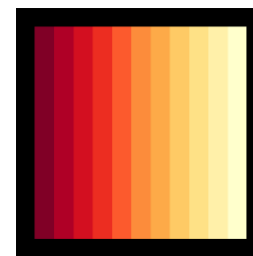
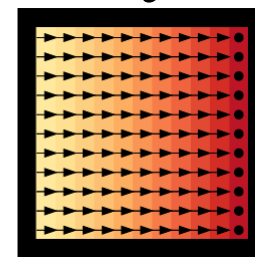
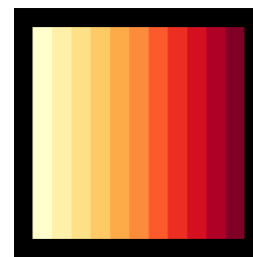


Gridworld representation

Pack all red

R

Q



Unpack all red

W1: Task Composition

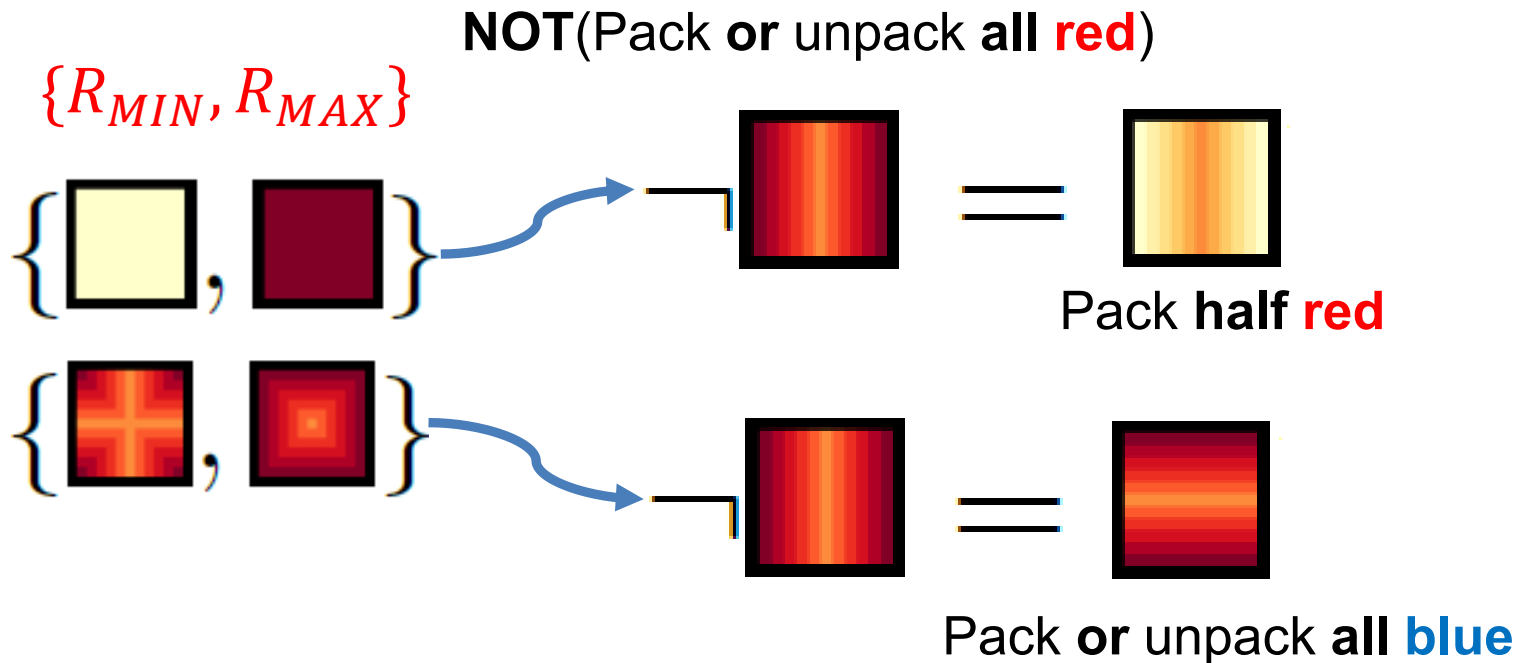
Logic Operators

- **OR:** $A \vee B := \max\{R_A(s, a, s'), R_B(s, a, s')\}$
- **AND:** $A \wedge B := \min\{R_A(s, a, s'), R_B(s, a, s')\}$
- **NOT:** $\neg A := (R_{MAX}(s, a, s') + R_{MIN}(s, a, s')) - R_A(s, a, s')$

Reward bounds

W2: Different Composition Semantics

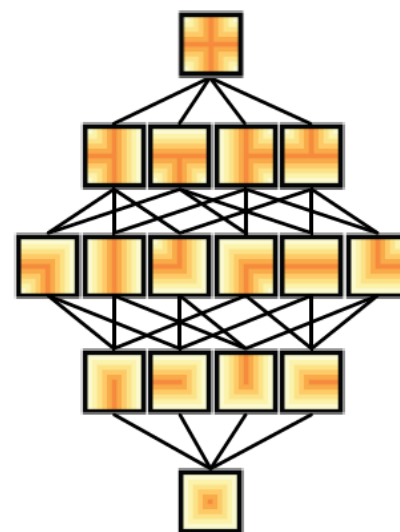
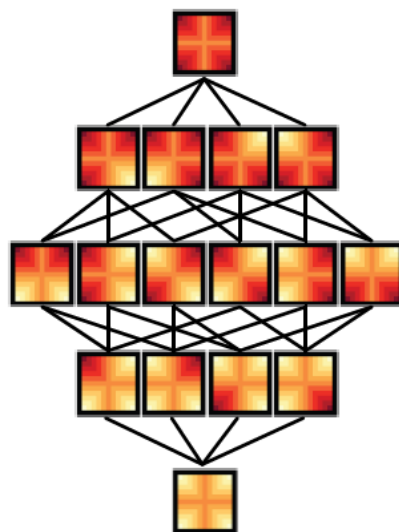
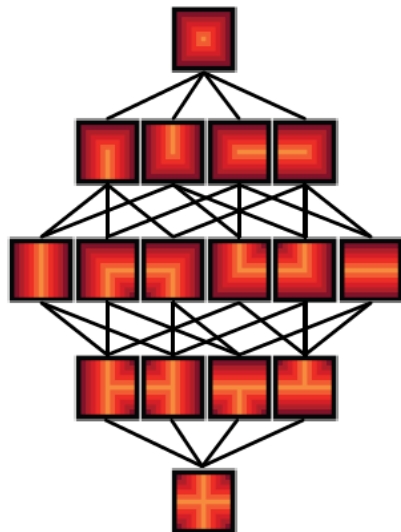
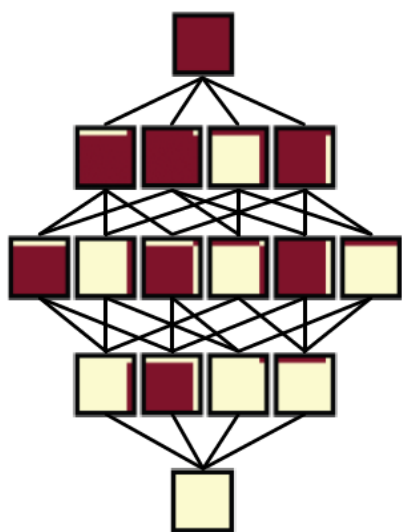
Bounds change semantics:



W3: Boolean Tasks with Dense Rewards

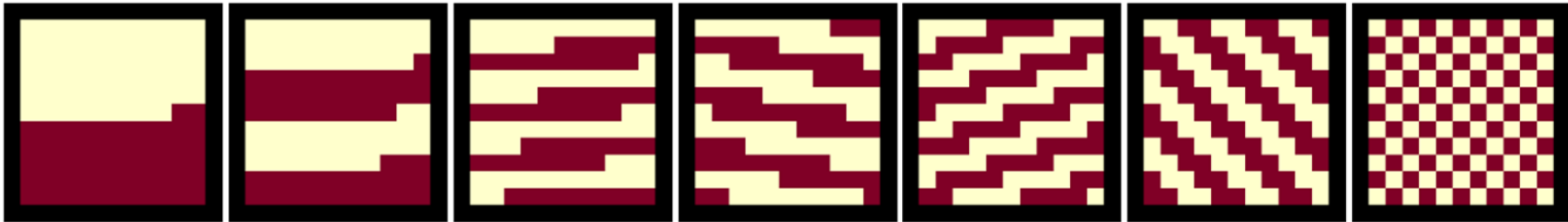
Given a basis: $\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\}$, $\{\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\}$, $\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\}$, $\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\}$

Sub spaces



W4: Construct a Basis

Bounds: {, 



W5: World Value Function

Intuition: while solving one task, we should **learn about other tasks** that we may need to solve in the future

$$\bar{r}(s, \mathbf{g}, a) = \begin{cases} \bar{r}_{MIN} & \text{if } \mathbf{g} \neq s \in G \\ r(s, a) & \text{otherwise} \end{cases}$$

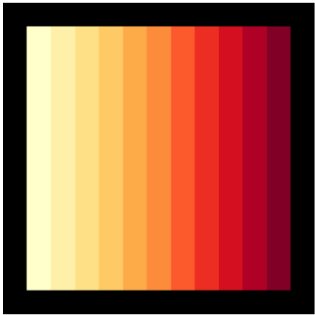
$$\bar{\pi}(s, a, \mathbf{g}) \rightarrow [0,1]$$

$$Q^{\bar{\pi}}(s, \mathbf{g}, a) = \mathbb{E} \left[\sum_{t=0}^{\infty} \gamma^t \bar{r}(s_t, \mathbf{g}, a_t) \right]$$

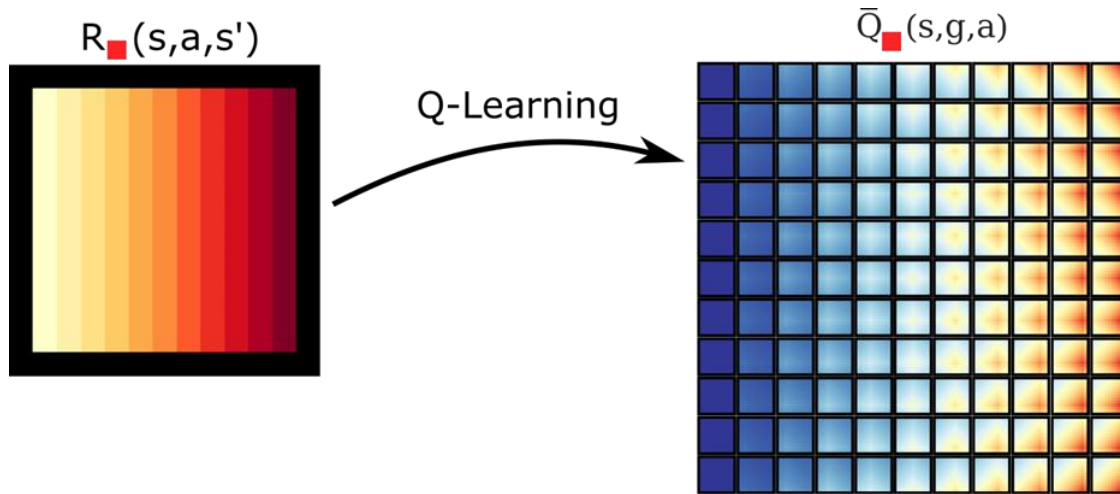
*Agent-chosen
goals
Mastery!!!
Purely driven by
reward maximization
No spoon-feeding*

W6: Mastery

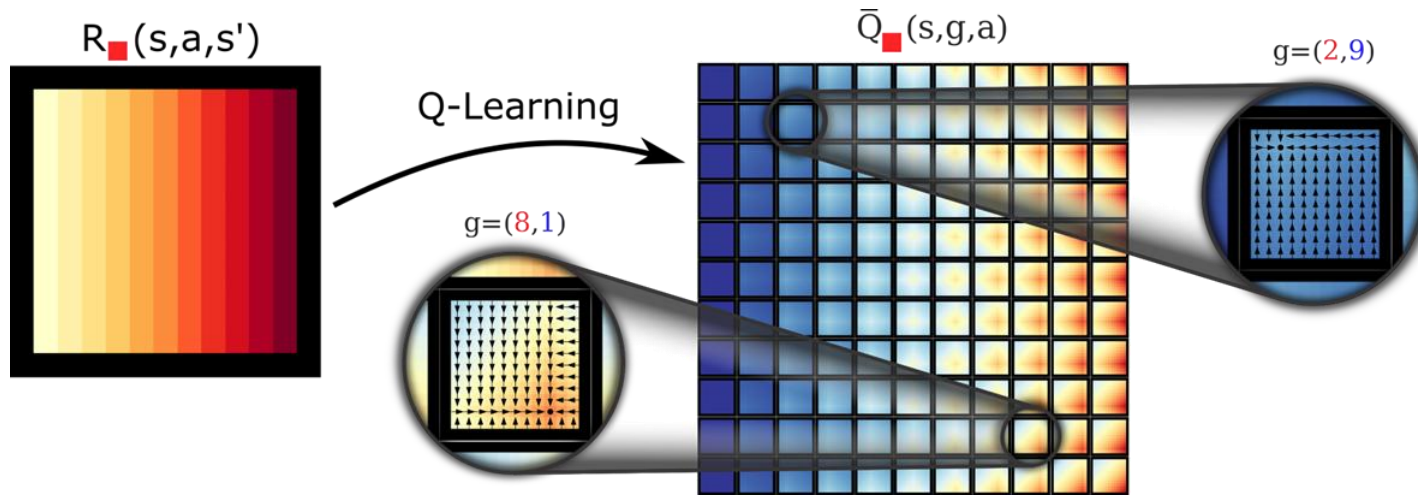
$R_{\blacksquare}(s,a,s')$



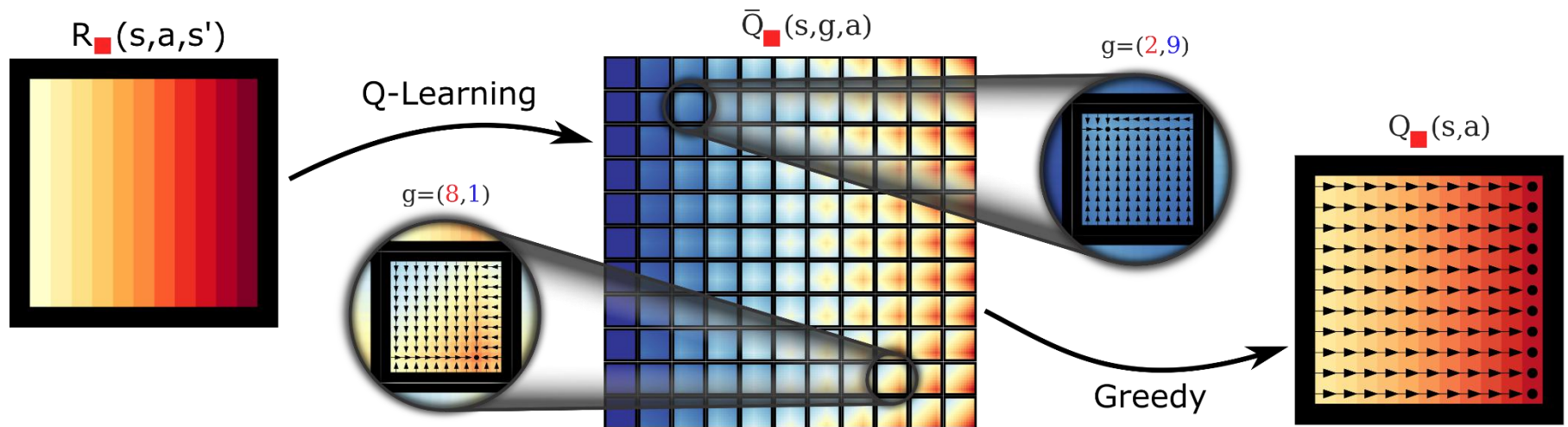
#6 Mastery



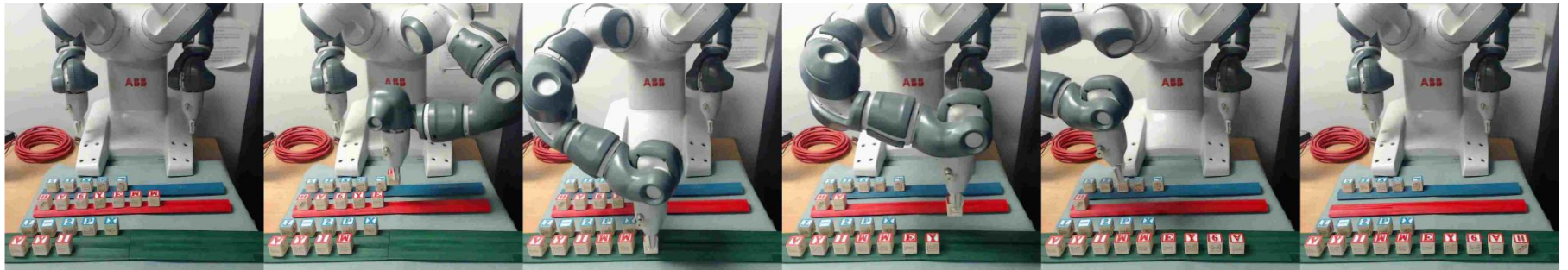
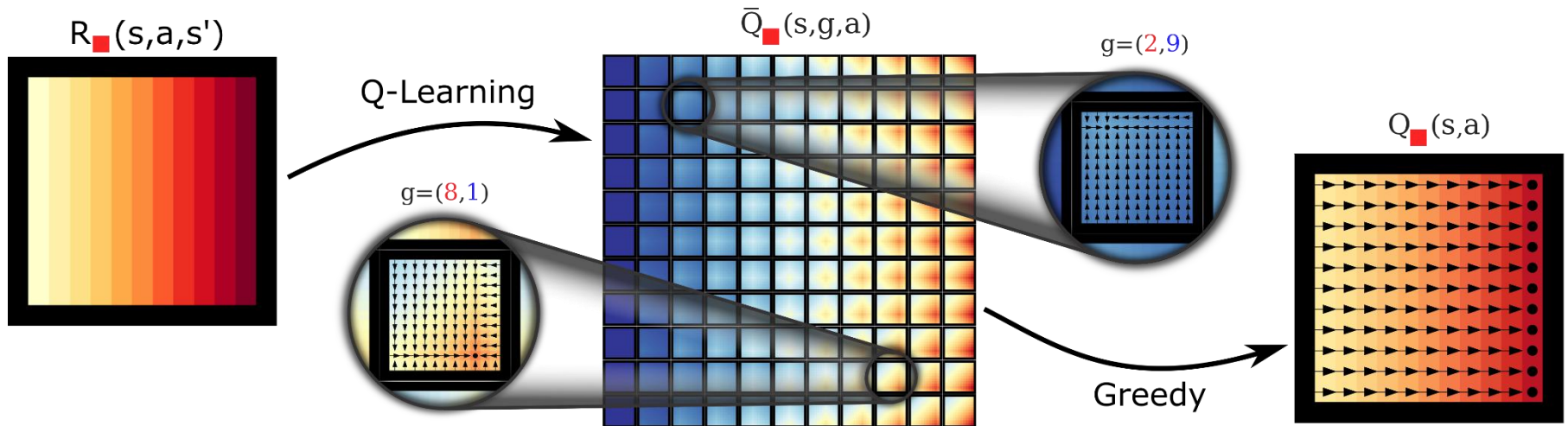
W6: Mastery



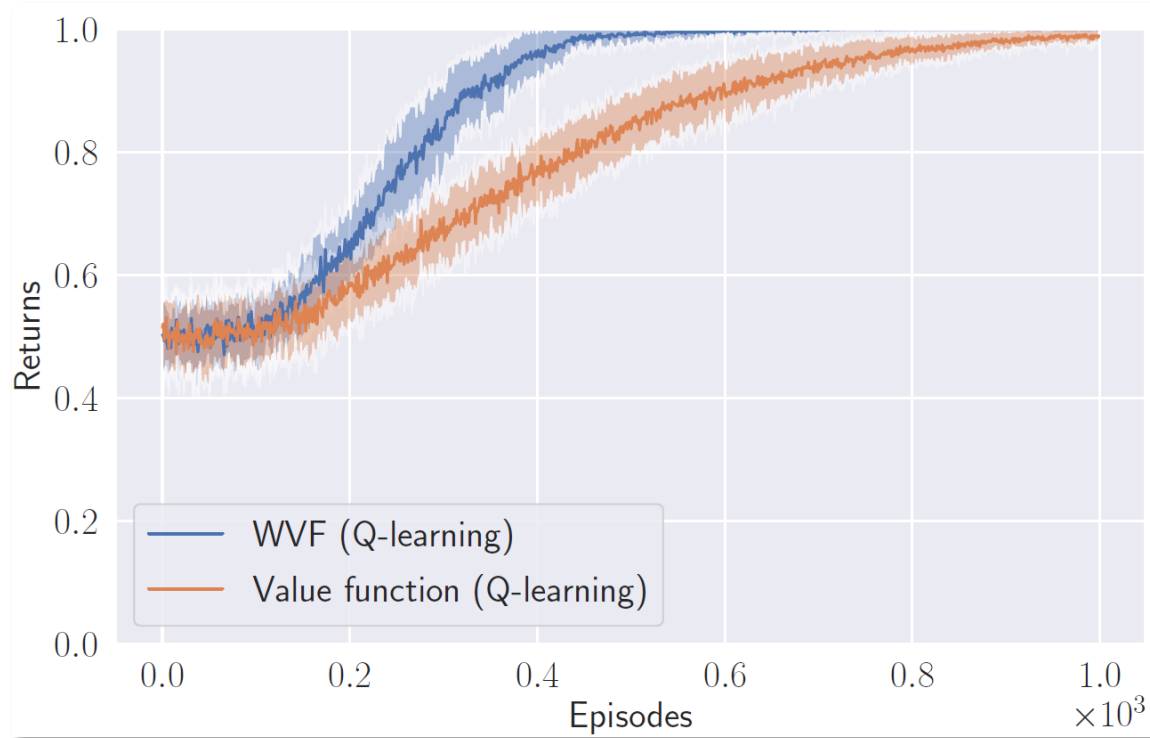
W6: Mastery



W6: Mastery



W7: Better sample-efficiency



W8: Skill Composition

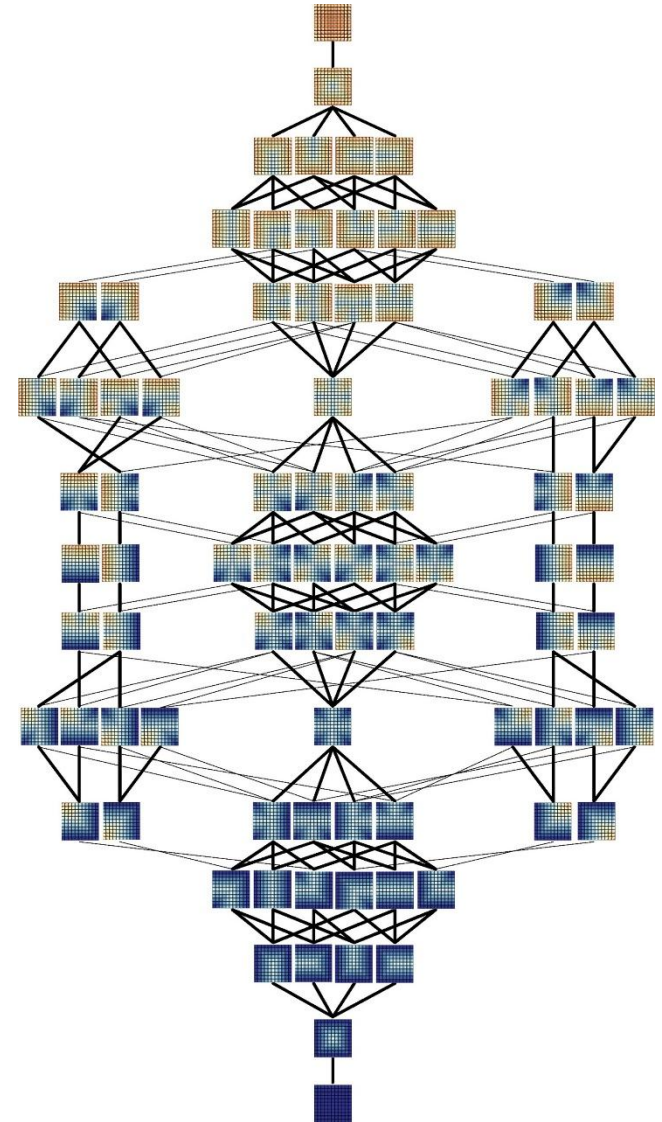
Logic Operators

- **OR:**
 - $Q_A \vee Q_B := \max\{Q_A(s, a, g), Q_B(s, a, g)\}$
- **AND:**
 - $Q_A \wedge Q_B := \min\{Q_A(s, a, g), Q_B(s, a, g)\}$
- **NOT:**
 - $\neg Q_A := (Q_{MAX}(s, a, g) + Q_{MIN}(s, a, g)) - Q_A(s, a, g)$

W9: Homomorphism over tasks and skills

Logic Operators

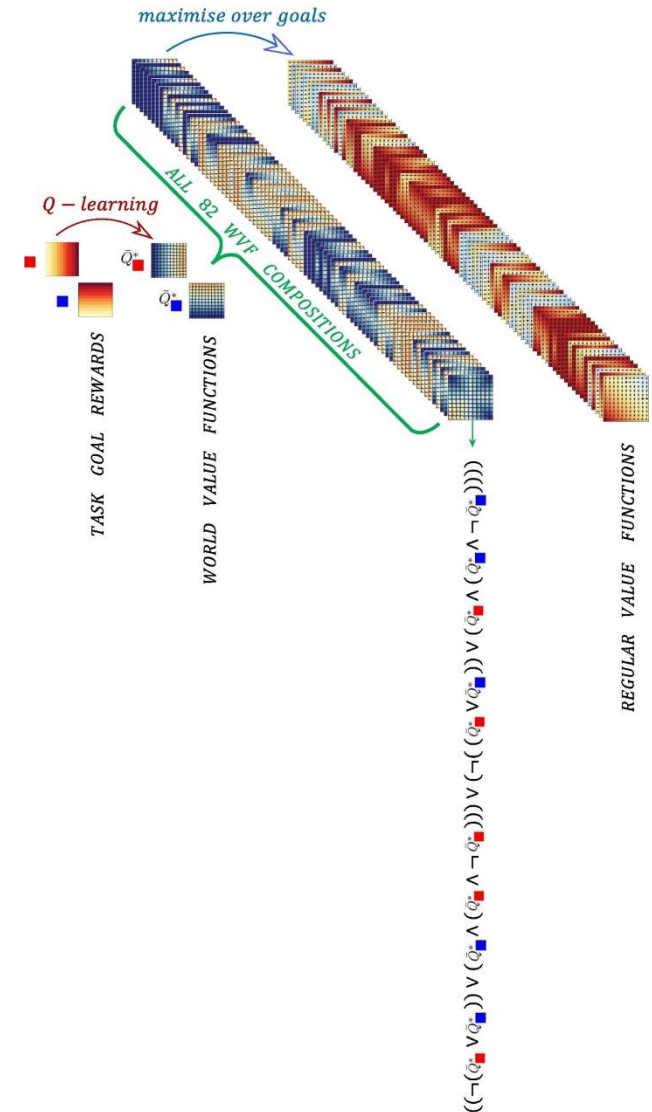
- **OR:**
 - $Q_A \vee Q_B := \max\{Q_A(s, a, g), Q_B(s, a, g)\}$
- **AND:**
 - $Q_A \wedge Q_B := \min\{Q_A(s, a, g), Q_B(s, a, g)\}$
- **NOT:**
 - $\neg Q_A := (Q_{MAX}(s, a, g) + Q_{MIN}(s, a, g)) - Q_A(s, a, g)$



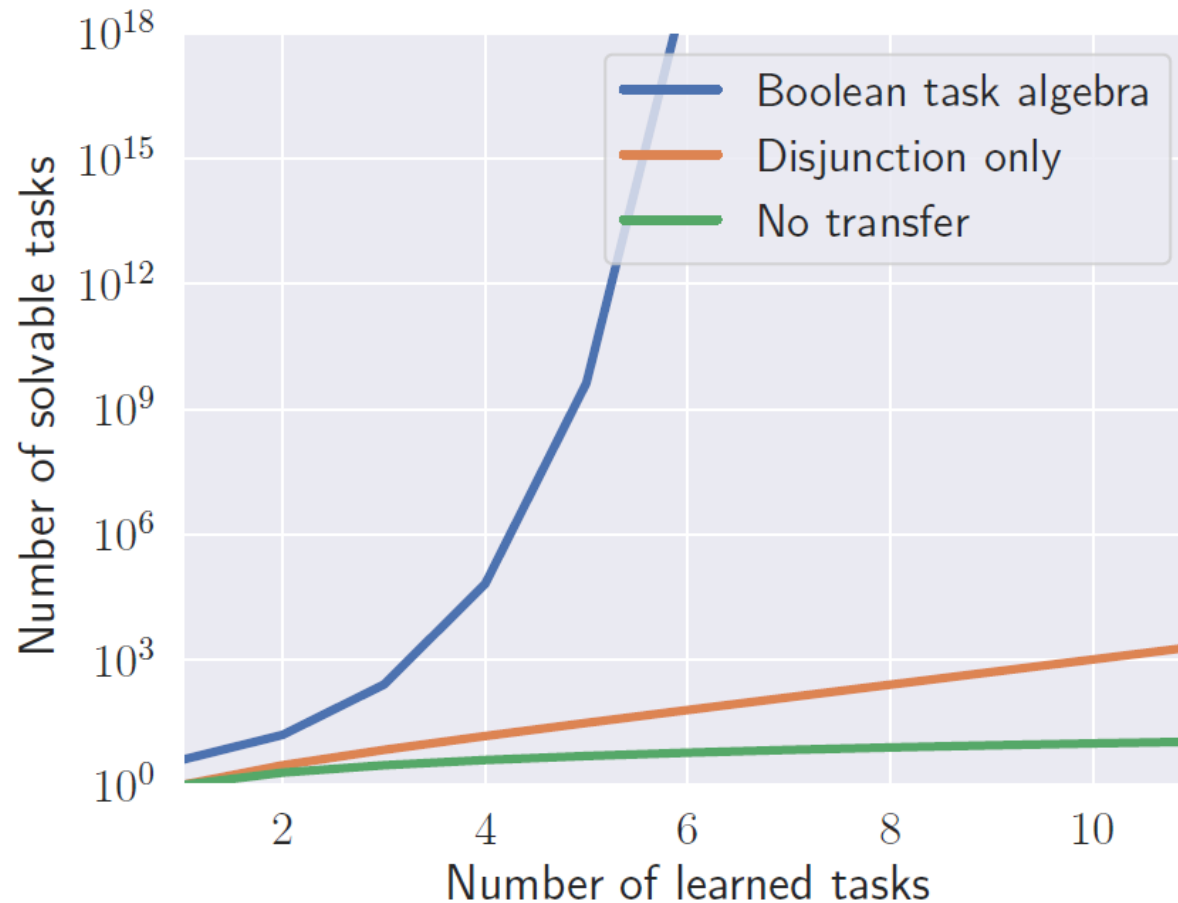
W10: Combinatorial explosion of skills

Logic Operators

- **OR:**
 - $Q_A \vee Q_B := \max\{Q_A(s, a, g), Q_B(s, a, g)\}$
- **AND:**
 - $Q_A \wedge Q_B := \min\{Q_A(s, a, g), Q_B(s, a, g)\}$
- **NOT:**
 - $\neg Q_A := (Q_{MAX}(s, a, g) + Q_{MIN}(s, a, g)) - Q_A(s, a, g)$

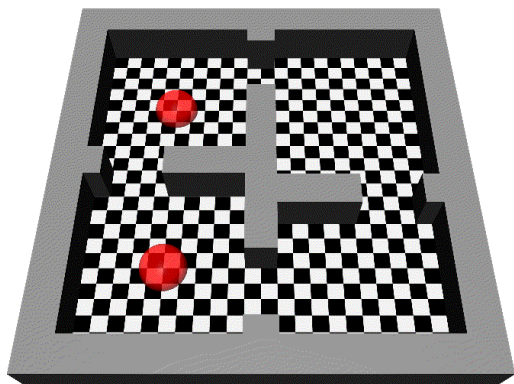


W11: Super-Exponential explosion of skills

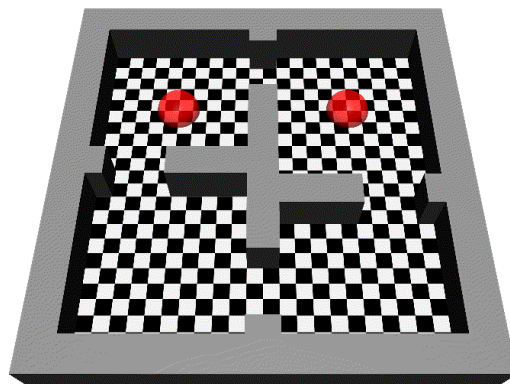


W12: Any RL Algorithm and Environment

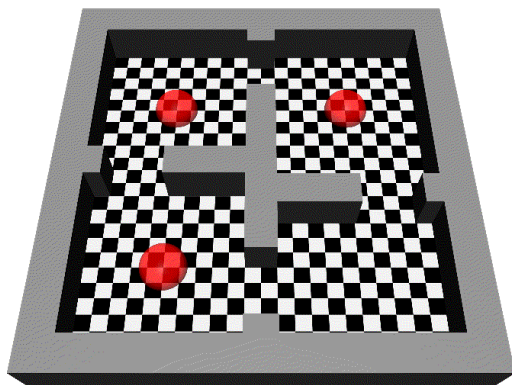
DQN/TD3/SAC/PPO/etc
Tabular/Discrete/continuous/etc



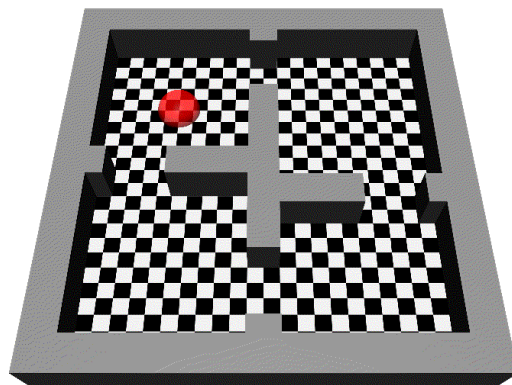
Skill: LEFT



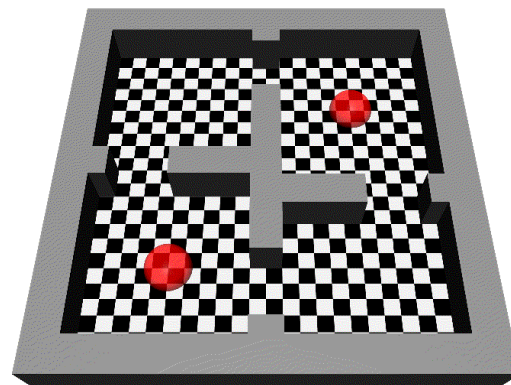
Skill: TOP



TOP OR LEFT



TOP AND LEFT



TOP XOR LEFT

And much more Ws!!!

Come chat at our poster

