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Markov need not apply! RL agents can efficiently handle long-term dependencies by learning what to remember, reducing memory and compute costs while preserving optimality.

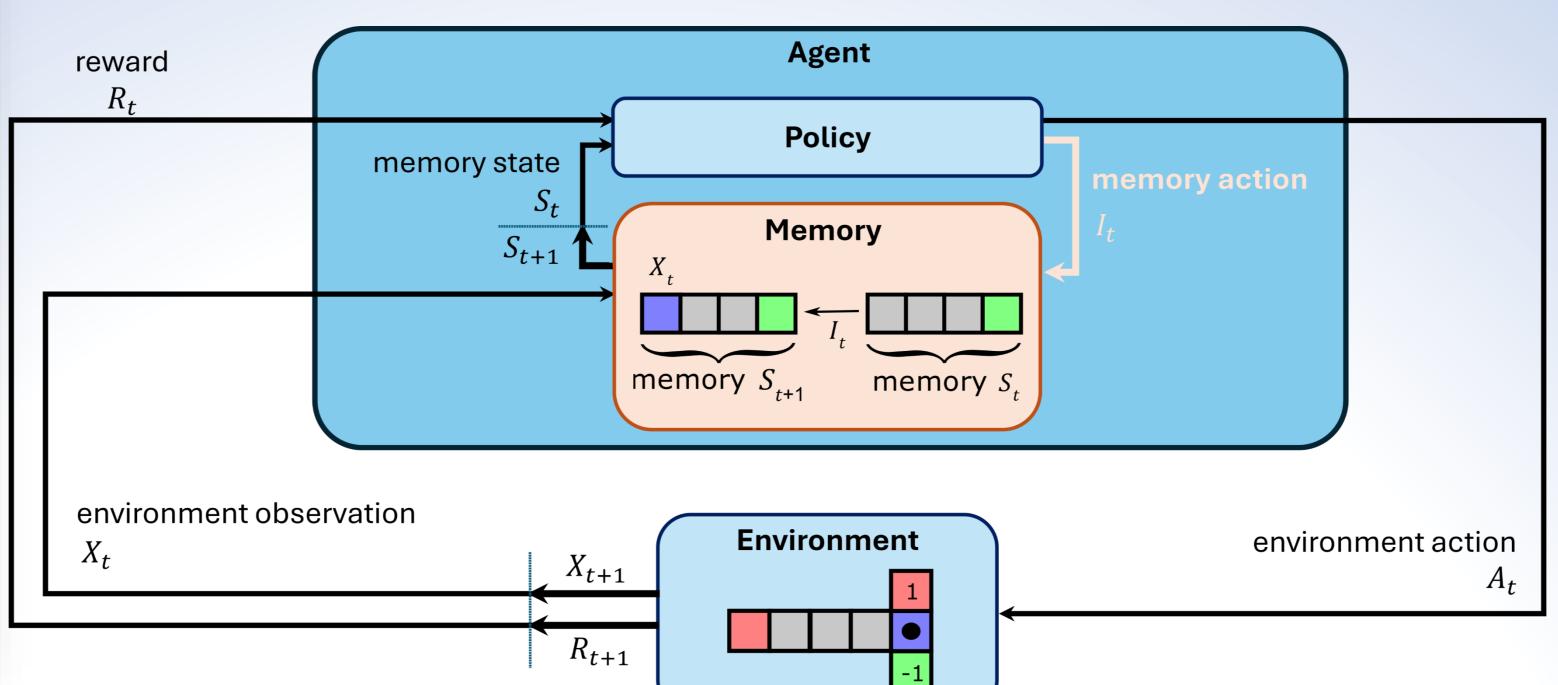
#### Motivation

Unlike a standard Frame Stack, which blindly retains recent observations (needs full history  $k^*$ ), we want agents that learn only the minimal number of observations  $\kappa$  to retain based only on their relevance for reward maximisation.

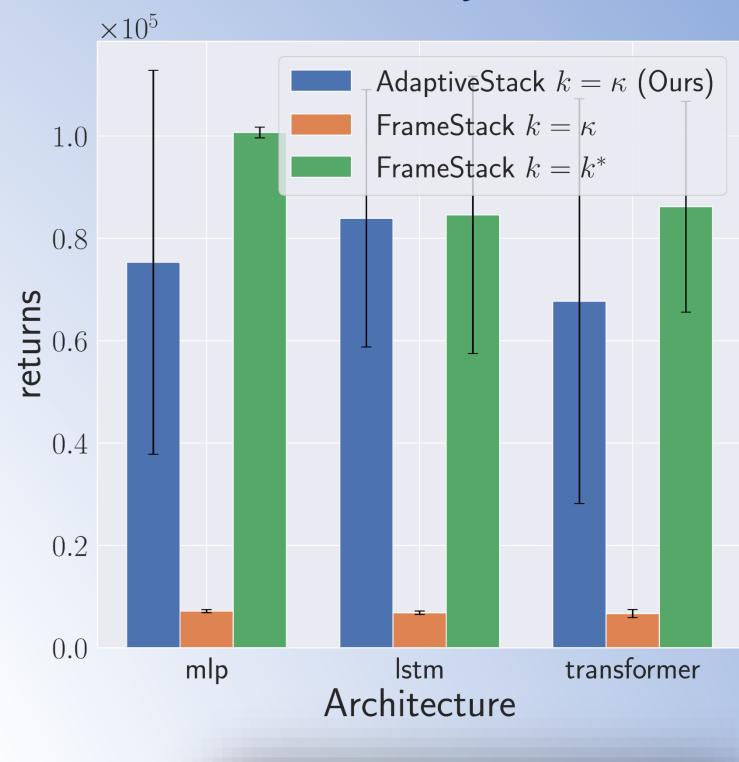
Frame Stack → **exponential increase** in observations, which impacts **compute c** and **memory w** 

Architecture	Memory Type	$ c _{a \sim \pi_{\theta}}$	$ c _{\mathrm{TD}}$	$ w _{a \sim \pi_{\theta}}$	$ w _{\mathrm{TD}}$
MLP or LSTM MLP or LSTM	Frame Stack Adaptive Stack	$\frac{\Omega(k^*)}{\Omega(\kappa)}$	$\frac{\Omega(k^*)}{\Omega(\kappa)}$	$\frac{\Omega(k^*)}{\Omega(\kappa)}$	$\frac{\Omega(k^*)}{\Omega(\kappa)}$
Transformer Transformer	Frame Stack Adaptive Stack	$\frac{\Omega(k^{*2})}{\Omega(\kappa^2)}$	$\frac{\Omega(k^*)}{\Omega(\kappa)}$	$\frac{\Omega(k^{*2})}{\Omega(\kappa^2)}$	$\frac{\Omega(k^*)}{\Omega(\kappa)}$

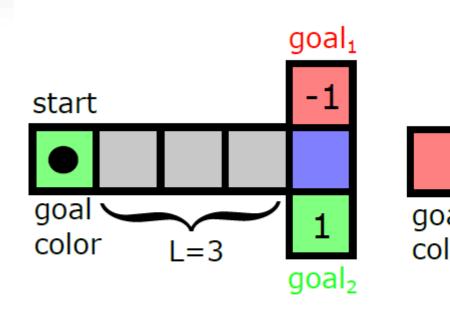
## Adaptive Stacking

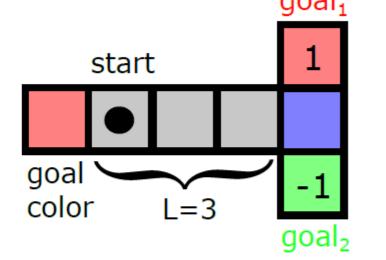


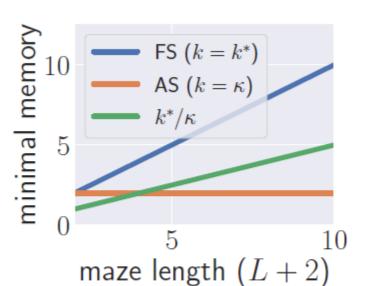
# Tmaze (Length=16)

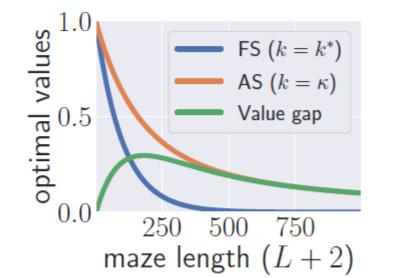


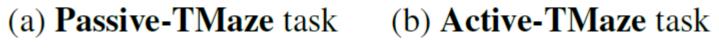
### RL with Internal Memory Decisions

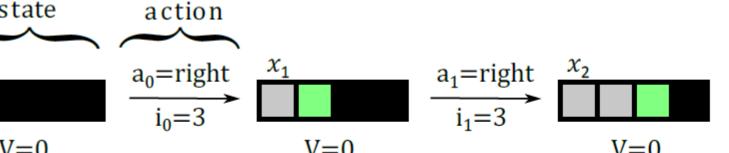


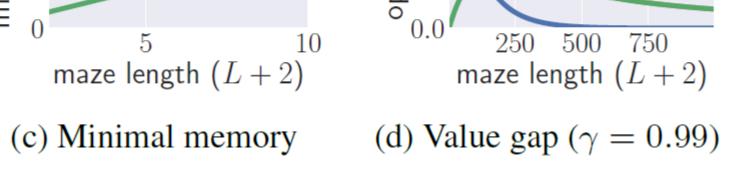


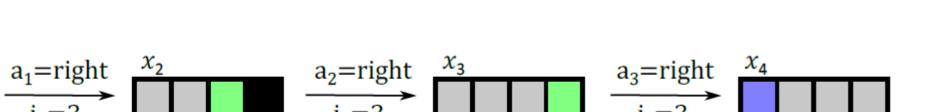




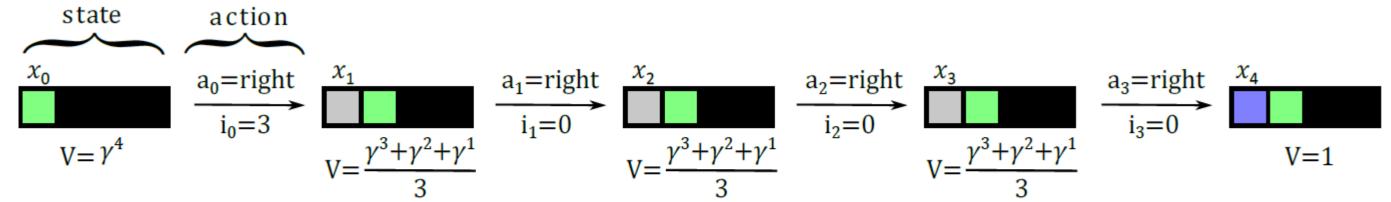




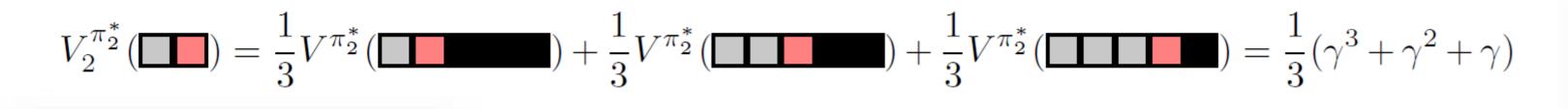




(a) **Frame Stacking**. At every time step, the agent pops the last observation in the memory stack in order free up space to push the new observation into the stack.



(b) **Adaptive Stacking**. At every time step, the agent chooses which observation in the memory stack to pop in order to free up space to push the new observation into the stack.



## Adaptive Stacking

Get initial observation  $x_0 \in \mathcal{X}$ Initialise observation stack  $s_0 \leftarrow [x_0]_k$ 

foreach timestep t = 0, 1, ..., T while episode is not done do

$$\langle a_t, i_t \rangle \leftarrow \begin{cases} \arg \max_{\langle a, i \rangle} Q(s_t, \langle a, i \rangle) & \text{w.p. } 1 - \varepsilon \\ \text{a random action} & \text{w.p. } \varepsilon \end{cases}$$

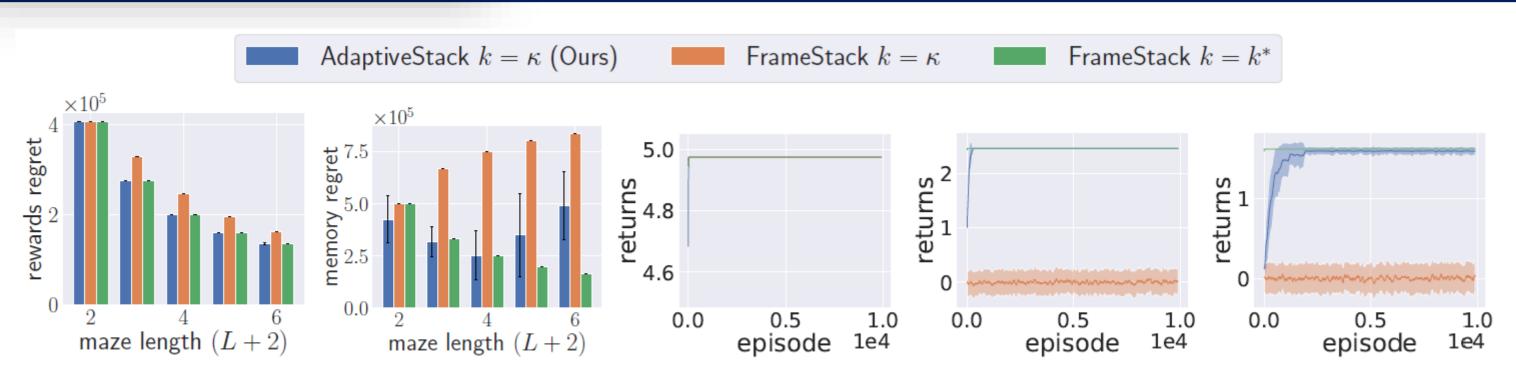
Execute  $a_t$ , get reward  $r_{t+1}$  and next observation  $x_{t+1}$ Remove observation from stack  $s_{t+1} \leftarrow pop(s_t, i_t)$ Push observation into stack  $s_{t+1} \leftarrow push(s_{t+1}, x_{t+1})$ 

**Remark 1** Uncertainty in history may harm value expectations,  $|V^*(x_{t:t-k^*}) - V_k^{\pi_k^*}(s_t)| > 0$ , but it does not necessarily harm policy optimality as long as the uncertain differences are irrelevant for optimal decision making:  $V^*(x_{t:t-k^*}) = V^{\pi_k^*}(x_{t:t-k^*})$ .

**Definition 1** Define  $\kappa$  to be the smallest memory length such that there exists a policy  $\pi_{\kappa}^*$  satisfying  $V^{\pi_k^*}(x_{t:t-k^*}) = V^*(x_{t:t-k^*})$  for all t.

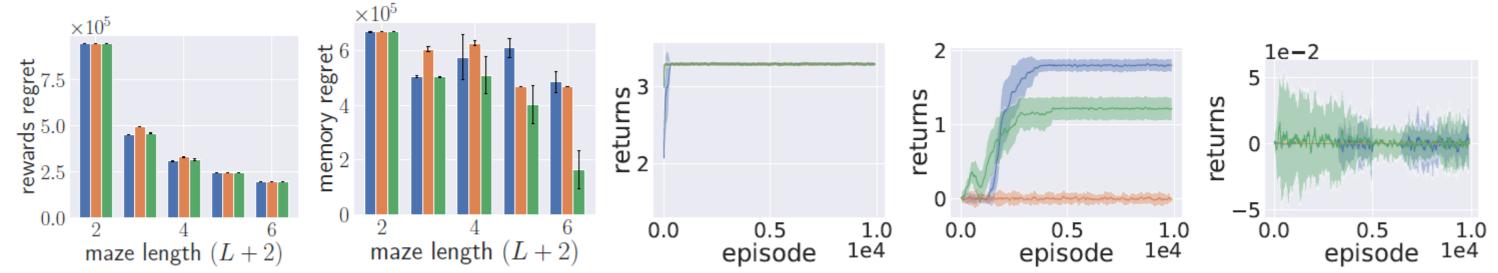
**Theorem 1** Let  $\mathbb{A}$  be an RL algorithm that converges under Frame Stacking with  $k \geq k^*$ . If  $\mathbb{A}$  uses unbiased value estimates to learn optimal policies, then it also converges under Adaptive Stacking with  $k \geq \kappa$  observations, assuming the policy class is sufficiently expressive.

## Training



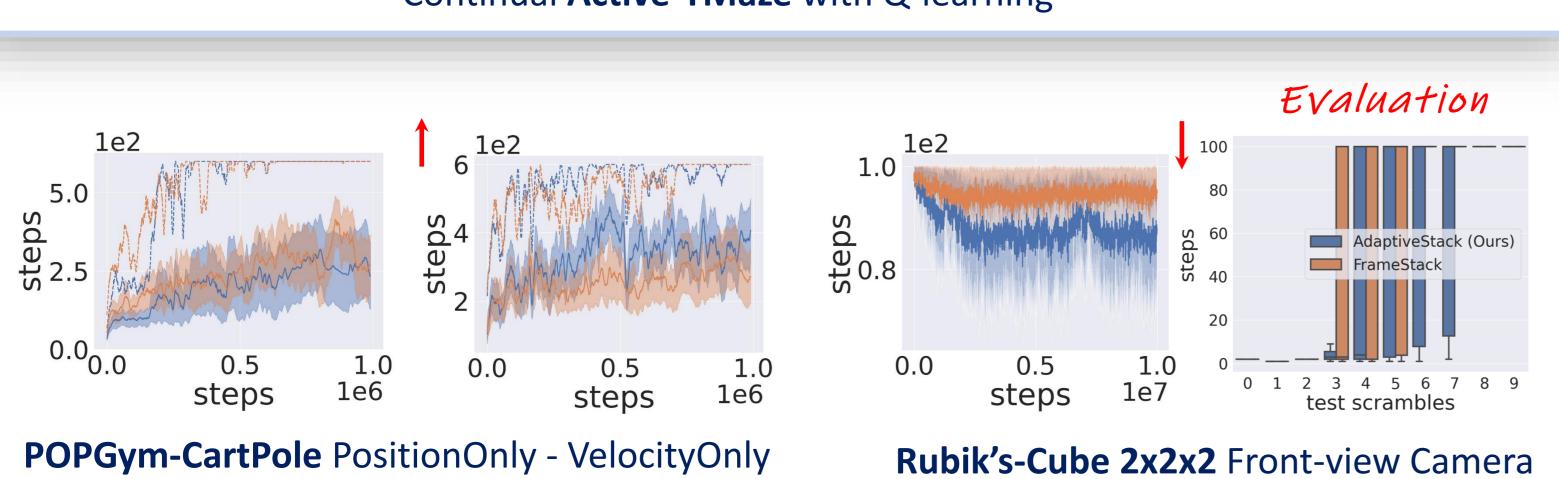
(a) Rewards regret (b) Memory regret (c) Returns L=0 (d) Returns L=2 (e) Returns L=4

### Continual Passive-TMaze with Q-learning



(a) Rewards regret (b) Memory regret (c) Returns L=0 (d) Returns L=2 (e) Returns L=4

### Continual Active-TMaze with Q-learning



Memory scaling

